Errata for Spectral Theory of Operators on Hilbert Spaces

p.5, line-17 (displayed formula). Replace "$\sum_{i=0}^{\infty}$" with "$\sum_{k=0}^{\infty}$"

p.9, lines 2,8. Replace twice "on" with "onto"

p.18, line 2. Replace "On" with "In"

p.20, line 8. Replace twice "T^*" with "T"

p.23, line 3. Replace "on" with "onto"

p.23, line 2. Inset "a" [...of a normed space...]

p.24, line 3. Replace "$W|_{N(W)^\perp}$" with "$W|_{\mathcal{N}(W)^\perp}$"

p.24, line -5. Replace twice. "$\dim(\mathcal{X}) < \infty$" with "$\dim \mathcal{X} < \infty$"

p.25, line 12. Replace "on" with "onto"

p.25, line -12. Replace "Theorem 5.73" with "Proposition 5.73"

p.33, line -9. Delete ")" [$\nu \in \mathbb{C}\backslash\sigma_{AP}(T)$...]

p.35, line 16. Replace "-1^{n+1}" with "$(-1)^{n+1}$"

p.42, line 10. Replace "$\langle Tx_n\, x_n\rangle \to \lambda$" with "$\langle Tx_n\,; x_n\rangle \to \lambda$"

p.43, line -10. Replace "$\lambda = \|T\|$" with "$|\lambda| = \|T\|$"

p.45, line -4. Replace "bases" with "basis"

p.49, lines 12,13 (displayed formulas). Replace twice "$k \geq 1$" with "$k \geq 0$"

p.49, line -5. Replace "$\bigoplus_{k=0}^{\infty} \mathcal{H}_k$" with "$\bigoplus_{k=-\infty}^{\infty} \mathcal{H}_k$"

p.50, lines -7,-1. Replace twice "stabilities" with "stability"

p.51, lines -7,-1. Replace "an" with "a" [a homomorphism]

p.51, line -2. Replace "$ST - TS$" with "$ST = TS$"

p.52, line -15. Insert ")" [6.14])...]

p.52, line 10. (displayed formula). Replace "$\sigma(T) = \{\Phi...$" with "$\sigma'(T) = \{\Phi...$"

p.58, lines 26,27 (Notes). Replace "[63, Problem 6.14]" with "[66, Problem 6.14]"

p.59, lines 6,9,12. Replace three times "$\sup_{k\geq n}$" with "$\sup_{k\geq n+1}$"

p.60, line -13. Replace "a" with "an" [an orthonormal]

p.63, line 18. Replace "compact" with "compact normal"

p.66, line -10 (displayed formula). Replace "$\overline{f(y,x)}$" with "$\overline{\langle Fy\,; x\rangle}$"

p.67, line 3. Replace "of π with respect to ν" with "of ν with respect to π"

p.67, line 12 (displayed formula). Delete "$= p(F^*, F)$"

p.67, line 16. Delete "$= p(F^*, F)$"

p.69, line -7 (displayed formula). Replace ''$\|Up\|_2^2$" with "$\|p\|_2^2$"

p.71, line 15. Inset ")" [in $\mathbb{C}$). Thus ...]

p.71, line -13. Insert "be" [— can be viewed as...]

p.71, line -10. Replace "is bounded" with " ; disjoint union"

p.71, line -5. Replace "disjoint countable unions" with "countable disjoint unions"

p.71, line -3. Replace "disjoint" with "countable" [any countable]

p.72, line 1. Replace "$\Lambda_j = \Omega_j$ so that" with "if $\Lambda_j = \Omega_j$, then"

p.72, lines 3 to 6. Replace the paragraph (4 lines) as follows:

Let $\varphi : \Omega \to \mathbb{C}$ be the function obtained from the identity functions $\varphi_\gamma : \Omega_\gamma \to \Omega_\gamma$ as follows. If λ lies in the disjoint union $\Omega = \bigcup_{\gamma\in\Gamma} \Omega_\gamma$, then $\lambda \in \Omega_\beta$ for some $\beta \in \Gamma$, and so set $\varphi(\lambda) = \varphi_\beta(\lambda) = \lambda$ for every $\beta \in \Gamma$ such that $\lambda \in \Omega_\beta$. Since $\Omega_\beta = \sigma(T|_{\mathcal{M}_\beta}) \subseteq \sigma(T)$, it follows that φ is bounded (i.e., $\varphi \in L^\infty(\Omega, \mu)$) with

p.72, line -7. Replace $W\psi(\lambda) = \psi_\beta(\lambda)$ with $(W\psi)(\lambda) = \psi_\beta(\lambda)$

p.72, line -3. Replace "$(\sum_{\gamma\in\Gamma} \varphi_\gamma\psi_\gamma)_\beta(\lambda)$" with "$(W \sum_{\gamma\in\Gamma} \varphi_\gamma\psi_\gamma)(\lambda)$"

p.73, line 10. Replace "by" with "compare with" [compare with Proposition...]

p.73, line 11. Replace ", and hence Ω *is bounded*." with ". Note that φ *is bounded*."

p.75. Statement of Corollary 3.14, line 4. Replace

"*The following assertions are pairwise equivalent*."

with

"*Consider the following assertions*."

p.75. Statement of Corollary 3.14, line 9. Replace

"*Moreover, if any of the above equivalent assertions holds true, then the con-*"

with

"*Claim*: (c) $\Longleftrightarrow$ (d) $\Longrightarrow$ (a) $\Longrightarrow$ (b). *Moreover, if* (c) *holds true, then the con-*"

p.75. Replace proof of Corollary 3.14 as follows:

Proof. By Theorem 3.11, T is unitarily equivalent to a multiplication operator M_φ on $L^2(\Omega, \mu)$ for some φ in $L^\infty(\Omega, \mu)$. Let U be a unitary transformation in $\mathcal{B}[L^2(\Omega, \mu), \mathcal{H}]$ such that $U^*TU = M_\varphi$. Thus ψ in $L^2(\Omega, \mu)$ is a cyclic (star-cyclic) vector for the normal operator M_φ if and only if $x = U\psi$ in $\mathcal{H}$ is a cyclic (star-cyclic) vector for the normal operator T (in fact, $\bigvee_n\{T^n x\} = U \bigvee_n\{M_\varphi^n \psi\}$ and $\bigvee_{m,n}\{T^n T^{*m} x\} = U \bigvee_{m,n}\{M_\varphi^n M_\varphi^{*m} \psi\}$). Corollary 3.13 says that (a) $\Longrightarrow$ (b), and the Remark preceding Corollary 3.13 shows that (c) $\Longleftrightarrow$ (d) $\Longrightarrow$ (a). Now if (c) holds, then according to part (a) in the proof of Theorem 3.11, μ is finite and $\varphi : \Omega \to \Omega$ is the identity function ($\varphi(\lambda) = \lambda$) on the compact set $\Omega = \sigma(T)$. A trivial induction shows that $M_\varphi^n \psi = \varphi^n \psi$ and $M_\varphi^{*m} \psi = \overline{\varphi}^m \psi$, and so $\bigvee_{m,n}\{M_\varphi^n M_\varphi^{*m} \psi\} = \bigvee_{m,n}\{\varphi^n \overline{\varphi}^m \psi\} = \bigvee_{m,n}\{\lambda^n \overline{\lambda}^m \psi\}$ for every $\psi \in L^2(\Omega, \mu)$ and λ in Ω. Since μ is finite, the constant function 1 ($1(\lambda) = 1$ for all $\lambda \in \Omega$) lies in $L^2(\Omega, \mu) \cap L^\infty(\Omega, \mu)$. Set $\psi = 1$ and let $P(\Omega)$ denote the set of all polynomials $p(\cdot, \overline{\cdot}) : \Omega \to \mathbb{C}$ such that $\lambda \mapsto p(\lambda, \overline{\lambda})$. Thus $L^2(\Omega, \mu) = P(\Omega)^- = \bigvee_{m,n}\{\lambda^n \overline{\lambda}^m\} = \bigvee_{m,n}\{\varphi^n \overline{\varphi}^m 1\} = \bigvee_{m,n}\{M_\varphi^n M_\varphi^{*m} \psi\}$ (cf. proof of Theorem 3.11(a)) so that $\psi = 1$ is a star-cyclic vector for M_φ. □

p.76, line 18. Insert ")" [... $= C(\sigma(T))$,...]

p.76, line -9. Delete "bounded"

p.77, line 13. Insert "]" [... ; $(f - ig)\rangle$] for every...]

p.77, line -8. Replace 'bounded" with "included in a disjoint union of $\sigma(T)$"

p.78, line 9. Replace 'bounded" with "nonempty"

p.78, line 20. Replace "$\sigma(T)$" with "Ω" [in Ω. Note that, for each λ...]

p.80, line 9. Delete ", and since $\cup(\varphi(\Lambda)^-) \subseteq \Lambda^-$,"

p.80, line 10. Replace twice "(Λ^-)" with "(Λ)"

p.80, line 13. Delete "(see [76, Theorem 1.12])"

p.84, last line. Insert "the" [... in terms of the orthonormal basis...]

p.85, line 5. Insert "in" [A *diagonal operator* in $\mathcal{B}[\ell^2_\Gamma]$...]

p.85, line 15. Replace "unitary" with "unitarily" [... between the unitarily...]

p.87, line -18. Replace "sequence" with "family" [... bounded family of...]

p.91, line -12. Replace "spaces" with "space" [(of the linear space $\mathcal{A}$ *onto*...]

p.92, item (e). Insert "unital" [... *a unital* C*-*algebra*...]

p.94, lines 18,19. Replace "$F(\Omega) = \mathbb{C}^\Omega$" with "$F(\Omega) \subseteq \mathbb{C}^\Omega$".

p.94, line 25. Replace "which is the case of" with
"for instance, if $F(\Omega)$ is the algebra of all complex-valued bounded functions on Ω equipped with"

p.95, line 7. Insert "a" [... is not a $*$-algebra ...]

p.95, line -6. Replace "$\mathcal{B}(\sigma(T))$" with "$B(\sigma(T))$" [... $\phi \in \mathcal{B}(\sigma(T))$...]

p.96, lines 3,4. Replace twice "$\mathcal{B}(\sigma(T))$" with "$B(\sigma(T))$"

p.97, line -10. Replace "$\leq \infty$" with "$< \infty$"

p.98, line 7. Replace "$\Lambda \in \Omega$." with "$\Lambda \in \mathcal{A}_\Omega$."

p.102, line 5. Replace "$\psi(M_\phi)$" with "$\psi(M_\varphi)$" [$\psi(M_\varphi) = M_\psi$]

p.102, line 13. Replace "Theorem 4.64" with "Theorem 3.64"

p.103, line 14. Insert "a" [... with a parameterization...]

p.106, line 5. Replace "then" with "them" [one of them]

p.109, line 9. Replace "$\int_\Gamma f(\lambda)$" with "$\int_\Gamma f(\lambda)\, d\lambda$"

p.109, line 20 (displayed formula; last symbol). Replace "λ." with "$d\lambda$."

p.109, line 24. Replace "$\int_\Gamma f(\lambda) = 0$" with "$\int_\Gamma f(\lambda)\, d\lambda = 0$"

p.109, line -2. Replace "$\Gamma \subset \Delta \cap \rho(T) \neq \varnothing$" with "$\Gamma \subset \Delta^- \cap \rho(T) \neq \varnothing$"

p.109, line -2. Replace " (nonempty, since Γ' and Γ'' are distinct). Orient..." with " If $\widetilde{\Delta}$ is nonempty, then orient..."

p.110, line 1. Replace "$\operatorname{ins} \widetilde{\Gamma} = \widetilde{\Delta}$" with "$(\operatorname{ins} \widetilde{\Gamma})^- = \widetilde{\Delta}^-$"

p.110, lines 5,6. Replace twice "$\cup$" with "$\cap$"

p.110, line -3. Replace "in" with "is"

p.111, lines 5,6,7,8,9.10,11,12. Replace eight times "$(\lambda - \nu)^{-1}$" with "$(\nu - \lambda)^{-1}$"

p.112, line -15. Replace "$\mathcal{B}[\mathcal{H}]$" with "$\mathcal{B}[\mathcal{X}]$" [... :$A(\sigma(T) \to \mathcal{B}[\mathcal{X}]$...]

p.113, line -5. Replace "$A(\sigma(T)))$" with "$A(\sigma(T))$"

p.113, line -3. Delete "and according to Corollary 2.12(a),"

p.114, line 6 (displayed formula). Replace "$\frac{1}{2\pi i}\Big\|$" with "$\Big\|\frac{1}{2\pi i}$"

p.114, line -2. Replace "$\mathcal{A}(\sigma(T))$" with "$A(\sigma(T))$"

p.115, lines 1,4. Replace "$\mathcal{A}(\sigma(T))$" with "$A(\sigma(T))$"

p.121, line 8. Replace displayed formula "$R_j(\lambda) = R_T(\lambda)E_{\Delta_j} = R_{TE_{\Delta_j}}(\lambda) = R_{T_j}(\lambda)$"

p.124, line 3. Replace. "$\dim(\mathcal{N}(\lambda I - T)) < \infty$" with "$\dim \mathcal{N}(\lambda I - T) < \infty$"

p.124, line 14.. Replace. "$\dim(\mathcal{R}(E_{\Delta_1})) < \infty$" with "$\dim \mathcal{R}(E_{\Delta_1}) < \infty$"

p.124, line -2. Delete "a" [for some positive integer...]

p.124, last line. Replace "$\mathcal{N}(\lambda I - T)^{n-1}$" with "$\mathcal{N}((\lambda I - T)^{n-1})$"

p.125, line 5. Replace "$\mathcal{R}(E_\Delta)$" with "$\mathcal{R}(E_\lambda)$"

p.125, line 6. Replace "Theorem 4.18" with "Theorem 4.20"

p.126, line 2. Insert "a" [on a Hilbert space]

p.126, line -3. Replace "$B_{\delta_0}(\lambda_0)\backslash\{0\} =$..." with "$B_{\delta_0}(\lambda_0)\backslash\{\lambda_0\} =$..."

p.127, line 12. Replace "sone" with 1 "some"

p.127, line 12. Replace "$k < n$" with "$k < -n$"

p.127, lines -2. Replace "$\mathcal{A}(\sigma(T))$" with "$A(\sigma(T))$"

p.127, last line. Replace "$\mathcal{A}(\sigma(T|_{\mathcal{R}(E_\Delta)}))$" with "$A(\sigma(T|_{\mathcal{R}(E_\Delta)}))$"

p.128, line 2. Replace "calculus" with "calculi"

p.128, line 3. Replace "coincides" with "coincide"

p.133, last line. Delete "." after $\square$

p.135, line 29. Insert ")" [... (see, e.g.,[66, Problem 2.17).]

p.137, line -5 (displayed formula). Insert ")" [... $\ominus(\mathcal{R}(T) + \mathcal{N}(S))^{\perp}$)]

p.138, lines -12,-11,. Swap in each line "$\mathcal{N}(S_+)$" with "$\mathcal{N}(S_+^*)$"

p.138, line -10. Swap "$\mathcal{F}_\ell \backslash \mathcal{F}_r$" with "$\mathcal{F}_r \backslash \mathcal{F}_\ell$"

p.140, lines 6,10. Replace twice "is" with "are" [... are such that...]

p.141, line -5. Delete ")" at the end of the line [... $SS^{-1} = I$ in $\mathcal{B}[\mathcal{R}(S)]$.]

p.143, line 8. Replace "it" with "is" [... is a consequence...]

p.144, line -13. Insert "the" [... in the unital Banach algebra...]

p.147, line 3. Replace "$(\lambda I - T)$" with "$\lambda I - T$"

p.147, lines 14,16. Replace twice "$\mathcal{B}_\infty[\mathcal{H}]/\mathcal{B}[\mathcal{H}]$" with "$\mathcal{B}[\mathcal{H}]/\mathcal{B}_\infty[\mathcal{H}]$"

p.148, line -7 (displayed formula). Insert "$= \sigma_e(T)\backslash\sigma_{re}(T)$," before "$\subseteq$"

p.148, line -4 (displayed formula). Insert "$= \sigma_e(T)\backslash\sigma_{\ell e}(T)$," before "$\subseteq$"

p.149, lines 13,14 (item (e)). Swap "$\sigma_{\ell e}(T)$" with "$\sigma_{re}(T)$"

p.149, line -1, and p.150 lines 1,2. Replace

". Recall that $\sigma_{\ell e}(T)$ and $\sigma_{re}(T)$ are closed in $\mathbb{C}$, and so $\sigma_{+\infty}(T)$ and $\sigma_{-\infty}(T)$ are open in $\mathbb{C}$. Moreover, the expressions for $\sigma_{+\infty}(T)$ and $\sigma_{-\infty}(T)$ and Corollary 5.12 ensure that these sets are subsets of $\sigma_e(T)$."

with

", by recalling that $\sigma_e(T) = \sigma_{\ell e}(T) \cup \sigma_{re}(T)$, and also recalling that $\sigma_e(T)$, $\sigma_{\ell e}(T)$, and $\sigma_{re}(T)$ are all closed in $\mathbb{C}$, and so $\sigma_{+\infty}(T)$ and $\sigma_{-\infty}(T)$ are open in $\mathbb{C}$."

p.150, line 10. Replace "$0 = \dim \mathcal{N}(\lambda I - T)$" with "$0 = \dim \mathcal{N}(\overline{\lambda} I - T^*)$"

p.150, line -12. Replace "$B_\varepsilon(\lambda)\backslash\{0\}$" with "$B_\varepsilon(\lambda)\backslash\{\lambda\}$"

p.151, line -2. Replace "σ_{P_4}" with "$\sigma_{P_4}(T)$"

p.152, line 13,17. Replace twice "$\sigma_{+\infty}(T)$, and $\sigma_{+\infty}(T)$" with "$\sigma_{+\infty}(T)$, and $\sigma_{-\infty}(T)$"

p.155, line -14. Insert "the" [... in the Calkin algebra...]

p.155, line -7. Replace "$\dim(R(E_\lambda)) < \infty$" with "$\dim \mathcal{R}(E_\lambda) < \infty$"

p.156, line -14. Replace "$\sigma_0(T) \cap \pi(T) = \varnothing$" with "$\tau_0(T) \cap \pi_0(T) = \varnothing$"

p.156, line -9,10. Replace "Theorem 5.19" with "Corollary 5.18"

p.157, line 2. Insert "the" [... a subset of the closed set...]

p.158, line 17. Insert subscript "i" in $_{i=1}$ [... $x = \sum_{i=1}^{n} \langle x\,;e_i\rangle e_i$...]

p.160, lines 17,18. Replace three times "$\lambda T - T$" with "$\lambda I - T$"

p.164, line -7 (Claim 1, line 1). Insert "Suppose $\mathcal{R}(T^n)$ is closed for every $n \in \mathbb{N}_0$."

p.165, line 1, Replace "or," with "(since $\mathcal{R}(T^n)$ is closed) or,"

p.165, line -11. Replace "apply Claim 3 " with "apply Claims 1 and 3"

p.166, lines -12,-11. Replace "(since $\{\mathcal{R}(S^n)\}$ is nondecreasing)."
with "(S surjective implies $\operatorname{dsc}(S) = 0$)."

p.167, line 13. Replace "$\mathcal{R}(T^n)$ " with "$\mathcal{R}(T^m)$"

p.169, line -3. Replace "$\pi(T)$" with "$\pi_0(T)$"

p.170, line 2. Replace "Corollary 5. 26" with "Corollary 5.26"

p.172, line 11. Replace "Also, By" with " Also, by"

p.176, line 25. Replace "$\Phi \in \mathcal{A}''$" with " $\Phi \in \widehat{\mathcal{A}}''$"

p.176, line 25. Replace "$\Phi \in \mathcal{A}''\backslash\mathcal{A}''_F$" with " $\Phi \in \widehat{\mathcal{A}}''\backslash\widehat{\mathcal{A}}''_F$"

p.176, line 26. Replace "$0 \notin \sigma(A^{-1})$" with "$0 \notin \sigma''(A^{-1})$"

p.176, line 26. Replace "$\Phi \in \mathcal{A}''_F$" with "$\Phi \in \widehat{\mathcal{A}}''_F$"

p.176, line 28. Replace "Φ in $\mathcal{A}''_F$" with "Φ in $\widehat{\mathcal{A}}''_F$"

p.176, line 30 (displayed formula). Replace twice "$\Phi \in \mathcal{A}''_F$" with "$\Phi \in \widehat{\mathcal{A}}''_F$"

p.177, line -10. Replace "ensure" with "ensures"

p.184, line -8. Replace "$B_\varepsilon(\lambda)\backslash\{0\}$" with "$B_\varepsilon(\lambda)\backslash\{\lambda\}$"

p.185, line -9. Delete "$\mathcal{W} \subseteq$" [$\mathcal{B} \subseteq \big\{T \in \mathcal{W}\colon\ \operatorname{asc}(T) < \infty \iff \operatorname{dsc}(T) < \infty\big\}$]

p.190. Correct reference [57] as follows:

57. T. Ichinose, *Spectral properties of tensor products of linear operators I*, Trans. Amer. Math. Soc. **235** (1978), 75–113.

p.194, Replace "full spectrum, 52, 87" with "full spectrum, 52, 88"

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